

## A NEW APPROACH OF THE MAIN LANDING GEAR EQUATIONS

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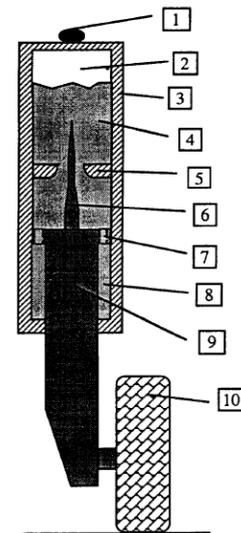
**Abstract:** In this paper an approach for modeling landing gear systems is presented. Specifically, a nonlinear model of an main landing gear is developed. This model includes nonlinear effects such as a polytropic gas law, velocity squared damping, a geometry governed model for the discharge coefficients, stick-slip friction effects and a nonlinear tire spring and damping model. An initial model was developed that only included the air-spring above the fluid, fluid dynamics through a fixed orifice, and a linear tire spring term.

**Key words:** *main landing gear, hydraulic spring, shock damper, linear model of equations*

### 1. Initial landing gear investigation

This chapter is intended to familiarize the reader with landing gear terminology and to demonstrate a mathematical development of the equations of motion for a telescoping landing gear. Figure 2-1 is intended to acquaint the reader with basic landing gear components. It shows the simplified components of a telescoping, main landing gear (as opposed to a nose gear).

Point 1 on the figure is a rigid body representation of the aircraft fuselage. Point 2 is a chamber containing compressed nitrogen which serves as a spring that carries the weight of the plane in ground operations. Point 3 refers to the main, upper cylinder which houses the compressed gas, hydraulic fluid, and within which the piston slides. Point 5 is the orifice plate. It is essentially a circular plate with a hole in the center through which the hydraulic fluid flows when the strut is stroking. It, along with the metering pin, point 6, controls the damping characteristics of the gear. Point 7 locates one of many rebound or snubber orifices. These holes lead into a small volume on the backside of the piston head (point 8) called the rebound or snubber chamber. The purpose of the snubber is to provide damping when the strut extends. The Point 9 is the piston.



**Figure 1.** *Schematic of typical telescoping main landing gear studied*

It houses the metering pin and is also the rigid connection of the wheel axle. Finally, point 10 is the tire. This element of the gear adds both spring and damping characteristics to the overall performance of the gear, and is selected carefully for various applications.

## 2. Nonlinear model development

This research discusses an independent development of a mathematical model of a main landing gear with all the relevant physical parameters included. The nonlinear equations of motion are developed for a telescoping main gear.

An initial model was developed that only included the air-spring above the fluid, fluid dynamics through a fixed orifice, and a linear tire spring term. This simple model allowed some trend comparison between the results of this model and the early results of the linearized gear. A metering pin was then added to change the main orifice effective diameter as a function of stroke. Another variation was the addition of a snubber, or rebound chamber. This feature provides damping while the gear is extending. The model includes constant seal friction as well as a variable friction that is a function of stroke. In a further effort to be realistic, a nonlinear tire model was added. This tire model has a spring rate that is a function of tire deflection and damping proportional to compression rate. In the equations developed below, the spring and damping coefficient are used as if they were constant. The nonlinear characteristics of each of these terms is included in the equations of motion that are actually integrated.

Figure 2 is a schematic of the gear used in the development of the equations of motion. This schematic is representative of a general telescoping-type main landing gear. It includes the aerodynamic lift on the plane, Lift, the upper mass (of the plane's fuselage) and the mass of the main cylinder lumped together as a rigid mass,  $M_u$ , and the mass of the piston and the mass of the tire, also lumped together as  $M_L$ . The inertial coordinate of the upper mass is  $X_{wg}$ . The zero value for  $X_{wg}$  is when the gear is fully extended with the tire just touching the ground. From this same gear configuration,  $X_a$ , the coordinate of the lower mass, is taken as zero at the axle of the tire. Therefore, when the gear is in some compressed state,  $X_a$  measures the deflection of the tire when the ground input,  $U(t)$ , is zero.

In the compressed nitrogen chamber (upper cylinder) with cross sectional area of  $A_u$  the pressure is  $P_u$ . Likewise, in the lower chamber with cross sectional area of  $A_L$ , there is a pressure of  $P_L$ . In the snubber chamber, with annulus area of  $A_R$ , the pressure is defined to be  $P_s$ . The orifice plate has a hole of

diameter  $D_{op}$  through which the metering pin, with variable diameter  $D_{pin}$  moves. Fluid reaches the snubber chamber through the orifices  $d_s^c$  and  $d_s^E$ , where the superscripts represent either the compression mode or extension mode respectively. The diameter of the piston,  $D_{pi}$ , is used to calculate  $A_R$ . Simply subtract the area of the piston shaft from that of the lower cylinder to get  $A_R$ . The tire is also shown in Figure 2 with a distinction of pointing out that the tire spring and damping coefficients,  $K_t$  and  $C_t$  are nonlinear and contribute to the calculation of the tire force  $F_t$ .

Figure 3 shows the forces acting on the upper mass. Balancing the forces on the upper mass gives the following equation:

$$M_u \ddot{X}_{wg} = M_u g - L - P_u A_u - P_L (A_L - A_O) + P_s A_R \pm f \quad (01)$$

The term on the left hand side of Eq. (01) is the inertial motion term,  $g$  is the gravitational acceleration,  $f$  is the friction present in the gear, and all other terms are as described previously. This equation assumes that the fluid pressure in the upper cylinder is identical to the pneumatic pressure.

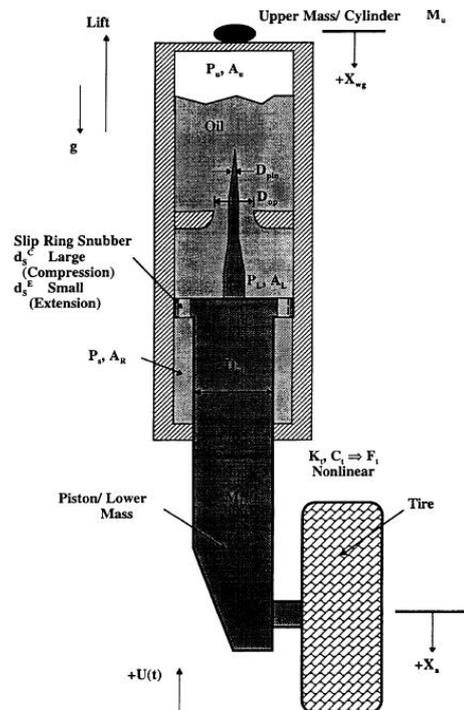


Figure 2 - Schematic of telescopic landing gear

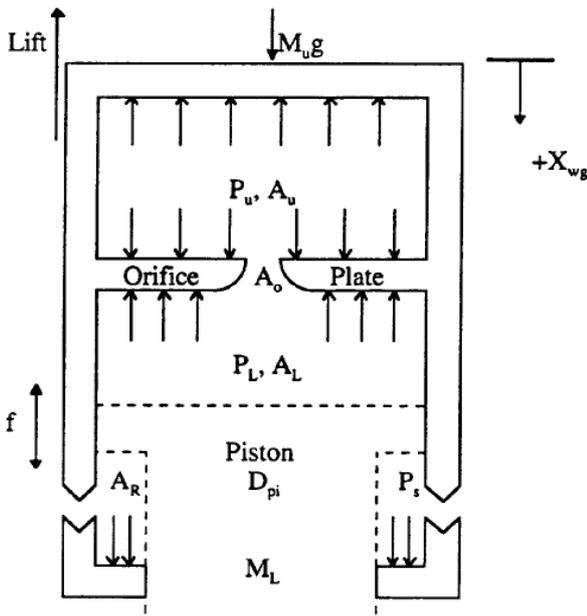


Figure 3 - Schematic of upper mass and main cylinder

In this area, reflects the fact that the metering pin is included, i.e. it is a variable cross-sectional

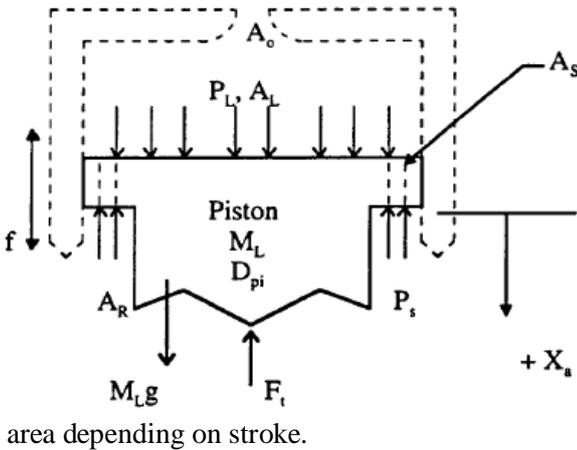


Figure 4 - Schematic of lower mass

Figure 4 shows the forces acting on the piston. Summing the forces on the lower mass (piston) the force balance equation is:

$$M_L \ddot{X}_a = M_L g + P_L(A_L - A_S) - P_S(A_R - A_S) - F_t \pm f \quad (02)$$

Where the left hand side of Eq. (02) is the inertial motion of the lower mass and  $A_s$  is the area of the snubber orifice.  $F_t$  is the force that is transmitted through the tire from the ground and

has the form:

$$F_t = K_t(X_a + U) + C_t(\dot{X}_U + \dot{U}) \quad (03)$$

where the tire force is a function of a nonlinear tire stiffness and a damping force that is composed of a damping coefficient that is proportional to the tire stiffness and the time rate of change of the tire deflection.

### 3. Relation of pressures to stroke position and stroke rate

The pressure terms in Eqs. (01) and (02) are as yet unknown and need to be related to the positional variables  $X_{wg}$  and  $X_a$  or their derivatives. The pressure of the compressed nitrogen in the upper cylinder can be described by the polytropic gas law for a closed system as:

$$P_U = P_{SI} \left( \frac{X_{SI}}{X_{Smax} - X_S} \right)^\gamma \quad (04)$$

where  $X_s$  is the stroke available, given by:

$$X_S = X_{Wg} - X_a \quad (05)$$

with  $X_{SI}$  as some initial length,  $P_{SI}$ , the charge pressure at  $X_{Smax}$ , and  $\gamma$ , the polytropic gas constant.  $X_{Smax}$  is the maximum value to which the gear can be extended. This form of representation of the pressure change is assumed to happen as a quasi-equilibrium process. The significance of the polytropic gas constant is that it describes the type of process that occurs. An average value is usually sufficient in application

Equation (04) was defined in such a manner that  $P_u$  will become very large when  $X_s$  is near  $X_{Smax}$ , i.e. the gear is nearly completely collapsed. This is a suitable representation of the process, with only the polytropic gas constant  $\gamma$  as an unknown.

The pressures ( $P_L$  and  $P_s$ ) of the fluid in the lower cylinder and in the snubber are related to the flow rates of the fluid into and out of those regions. The volumetric flow rates through the orifice plate hole,  $Q_c$ , and the snubber orifices,  $Q_s$ , can be determined by combining the continuity

equation and Bernoulli's equation for fluids. Flow is always from the higher pressure to the lower pressure. Bernoulli's equation for an incompressible fluid states that along a streamline,

$$\frac{P}{\rho} + \left(\frac{1}{2}\right)V^2 + Z = \text{const.} \quad (06)$$

where  $P$  is the pressure at some point,  $g$  is the gravitational acceleration,  $V$  is the velocity of the flow,  $\rho$  is the specific weight of the fluid which is equal to the fluid density ( $\rho$ ) multiplied by the gravitational acceleration ( $g$ ), and  $Z$  is the height difference from some zero reference. This equation assumes that the viscous effects within the fluid are negligible, the flow to be steady and incompressible, and that the equation is applicable along a streamline. Equating Bernoulli's equation (Eq. (06)) at two points in the flow along the same streamline yields:

$$\frac{P_1}{\rho} + \left(\frac{1}{2}\right)V_1^2 + Z_1 = \frac{P_2}{\rho} + \left(\frac{1}{2}\right)V_2^2 + Z_2 \quad (07)$$

In the case of a landing gear, the potential distance between  $Z_1$  and  $Z_2$  can be neglected as the distances involved are very small compared to the other terms. Equation (07) with the continuity equation for incompressible fluids which states  $Q = A_1 V_1 = A_2 V_2$  allows for the solution of this equation in terms of one of the velocities. Assuming that  $P_1 > P_2$ , i.e. the flow is from  $P_1$  to  $P_2$ , then solve for  $V_1$  from the continuity equation as:

$$V_1 = \frac{D_2^2}{D_1^2} V_2 \Rightarrow V_2 = \pm \sqrt{\frac{2(P_1 - P_2)}{\rho \left(1 - \frac{D_2^4}{D_1^4}\right)}} \quad (08)$$

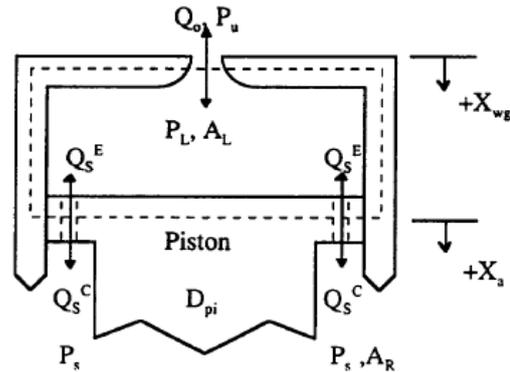
When the flow reverses, i.e.  $P_1 < P_2$ , then the velocity at point 2 is described by the above equation with the pressure terms switched and a negative sign on the square root. The ideal volumetric flowrate ( $Q_{\text{ideal}}$ ) for an incompressible fluid can be expressed as  $Q_{\text{ideal}} = A \cdot V$ . Now we have:

$$Q_{\text{real}} = C_d Q_{\text{ideal}} = A C_d V \quad (09)$$

Substituting Eq. (08) into Eq. (09) for velocity:

$$Q_{\text{real}} = A C_d \sqrt{\frac{2}{\rho \left(1 - \left(\frac{D_1}{D_2}\right)^4\right)}} \sqrt{P_1 - P_2} \rightarrow P_1 > P_2 \quad (10)$$

For our landing gear, there are two flows that are of concern, the flow through the orifice plate and the flow into and out of the snubber chamber. Define  $Q_s^C$  as the flow rate into the snubber chamber in the compression mode, where the snubber orifice area ( $A_s$ ) becomes  $A_s^C$ , which allows larger flow. The flow rate through the snubber orifice during the extension mode is defined as  $Q_s^E$ , and the area  $A_s$  becomes  $A_s^E$ , which only allows small, restricted flow. In both cases, the flow through the main orifice plate is  $Q_o$ .



**Figure 5** - Control volume between piston and orifice plate

Figure 5 shows the direction of fluid flow into and out of a control volume in the lower chamber as a function of stroke mode (extension or compression). In relating the flow rates to the pressures, defining a control volume as shown by the dashed line in Figure 5 is necessary. The stroke rate is defined as

$$\dot{X}_s = \dot{X}_{wg} - \dot{X}_a \quad (11)$$

where the compression mode is given by  $X_s > 0.0$ , and the extension mode by  $X_s < 0.0$ . The flow is assumed to be negative leaving the control volume, and is positive entering it. For an incompressible fluid, the volumetric flow rates for compression and extension can be written as:



Additional information about the flow rate-pressure relationship can be gained by studying a control volume in the snubber chamber as shown by the dashed line in Fig.6 .

The variables  $A_R$  and  $D_R$  in Fig.6 are the rebound chamber annulus area and effective diameter respectively.  $P_s$  is the pressure in the rebound chamber and  $d_s^c$  and  $d_s^E$  are the diameters of the snubber orifices in the compression mode and extension mode respectively. In the case of compression, where  $X_s > 0.0$  and  $P_L > P_s$ ,

$$Q_s^c + A_R \dot{X}_s = 0.0 \quad (18)$$

Substituting the flow rate  $Q_s^c$  of Eq. (15) into Eq.

$$-A_s^c C_{ds}^c \sqrt{\frac{2}{\rho(1-(\frac{d_s^c}{D_L})^4)}} \sqrt{P_L - P_s} + A_R \dot{X}_s = 0.0$$

(18) yields: (19)

From previous notation of  $E_i$  this expression

$$\text{becomes: } -E_2 \sqrt{P_L - P_s} + A_R \dot{X}_s = 0.0 \quad (20)$$

Rearrange Eq. (20) to get an expression for the pressures in terms of the stroke rate as:

$$\sqrt{P_L - P_s} = \frac{A_R}{E_2} \dot{X}_s \quad (21)$$

$$\Rightarrow P_L = P_U + \left(\frac{A_L - A_R}{E_1}\right)^2 \dot{X}_s^2 \quad (22)$$

where  $P_U$  is given in Eq. (04). Square both sides of Eq. (21) and solve for  $P_s$  as:

$$P_s = P_L - \left(\frac{A_R}{E_2}\right)^2 \dot{X}_s^2 \quad (23)$$

Similarly, for the extension case with  $X_s < 0.0$ :

$$P_L = P_U - \left(\frac{A_L - A_R}{E_3}\right)^2 \dot{X}_s^2 \quad (24)$$

$$P_s = P_L - \left(\frac{A_R}{E_4}\right)^2 \dot{X}_s^2 \quad (25)$$

These known pressures [Eqs. (04), (22), (23), (24), (25)] can now be substituted into Eqs. (23) and (24). Algebraic simplification of these equations leads to the compression and extension cases in terms of readily measurable quantities as:(01a)(02a)(01b)

$$M_U \ddot{X}_{wg} = M_U g - L + (A_R - A_L) P_{SI} \left(\frac{X_{SI}}{X_S}\right)^\gamma + \left\{ \left[ \left(\frac{A_L - A_R}{E_1}\right)^2 - \left(\frac{A_R}{E_2}\right)^2 \right] A_R - \left(\frac{A_L - A_R}{E_1}\right)^2 (A_L - A_0) \right\} \dot{X}_s^2 + f$$

$$M_L \ddot{X}_a = M_L g - (A_L - A_R) P_{SI} \left(\frac{X_{SI}}{X_S}\right)^\gamma + \left\{ \left[ \left(\frac{A_L - A_R}{E_3}\right)^2 - \left(\frac{A_R}{E_4}\right)^2 \right] A_R - \left(\frac{A_L - A_R}{E_3}\right)^2 (A_L - A_0) \right\} \dot{X}_s^2 - F_t \pm f$$

$$M_U \ddot{X}_{wg} = M_U g - L + (A_R - A_L) P_{SI} \left(\frac{X_{SI}}{X_S}\right)^\gamma + \left\{ \left(\frac{A_L - A_R}{E_3}\right)^2 (A_L - A_0) - \left[ \left(\frac{A_L - A_R}{E_3}\right)^2 - \left(\frac{A_R}{E_4}\right)^2 \right] A_R \right\} \dot{X}_s^2 - f \quad (02b)$$

Introduce a new notation using subscripts to simplify the above equations: "1" and "2" will be associated with compression (equation set (a)), and "3" and "4" with extension (set (b)). With this change, the equations can be written in the form:

$$M_U \ddot{X}_{wg} = M_U g - L + C_{1/3} \dot{X}_s^2 + K_{1/3} X_S^{-\gamma} \pm f$$

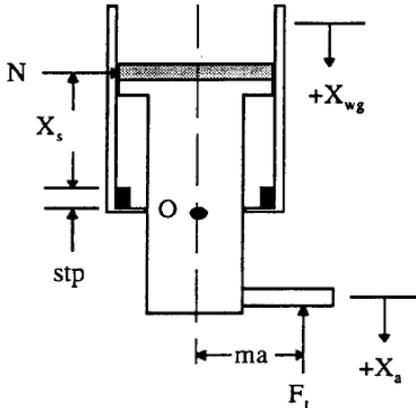
$$M_L \ddot{X}_a = M_L g + C_{2/4} \dot{X}_s^2 + K_{2/4} X_S^{-\gamma} - F_t \pm f \quad (01c)$$

$$M_L \ddot{X}_a = M_L g + C_{2/4} \dot{X}_s^2 + K_{2/4} X_S^{-\gamma} - F_t \pm f \quad (02c)$$

where the coefficients of the stroke rate squared term are assigned the  $C_i$ 's, and the coefficients of the stroke position term are the  $K_i$ 's.

The only unknown term left in these equations is friction. As mentioned previously, friction in this gear comes mainly from two sources, friction due to tightness of the seal and friction due to the offset wheel (moment). The seal friction is assumed to be a maximum value statically and some function of velocity in the dynamic state. The functional relationship between frictional force level and velocity could be determined through testing. The friction due to the offset wheel is the result of the moment produced by the nonaxially loaded piston within the cylinder.

It can be seen from Fig.7 that the force between the piston head and the cylinder,  $N$ , is a result of the tire force,  $F_t$ , applied at moment arm,  $ma$ , from the centerline of the piston. The frictional force due to the offset wheel ( $F_{ow}$  is assumed to be of the form (refer to Fig.7):



**Figure 7** - Schematic of gear for friction model development

$$F_{ow} = \mu N \quad (26)$$

Where  $N$  is the normal force of the cylinder wall resisting the side of the piston head, and  $\mu$  is the coefficient of friction between the two parts. To find the unknown force  $N$ , sum the moments about point  $O$  to zero to get:

$$\sum M_o : F_t ma - N(X_s + stp) = 0 \quad (27)$$

Where  $stp$  is the minimum distance between the piston head and the lower seal when the gear is fully extended. Rearrange Eq. (27) by isolating  $N$ , and then substitute  $N$  into Eq. (26) to get an explicit form of  $F_{ow}$ :

$$N = \frac{ma * F_t}{X_{wg} - X_a + stp} \quad (28)$$

$$F_t = \mu \left( \frac{ma * F_t}{X_{wg} - X_a + stp} \right) \quad (29)$$

The total friction in the landing gear,  $f$ , in equations (01c) and (02c) is now assumed to be:

This paper assumes that a proportionate part of the fuselage (half of the 80% of the total weight that rests upon the main gear) is treated as a lump mass centered at the centerline of the main upper cylinder. Also, this model takes into account only vertical loads on the strut. The tire is modeled as a nonlinear spring and damper. This tire model does not take into account spinning stiffness (because the test tire does not spin) or spin-up drag. The fluid is assumed to be incompressible and all structural members are assumed to be rigid, with each having only a vertical degree of freedom. These assumptions are good only for straight-line taxiing over runway profiles and landing impact (spin-up drag on the tire does not significantly effect the vertical loads on the strut). Any braking or turning maneuvers are not covered in the development. The equations developed here are the basis for a "rollout" simulation.

#### 4. Conclusions

In this, the nonlinear equations of motion were developed for a general, telescoping main landing gear.

These equations contain a pneumatic spring that is determined based on the polytropic gas compression law, a hydraulic damping that is proportional to the stroke rate squared, gravitational forces, lift, inputs from a runway, and finally friction, which is composed of both a constant seal friction and a variable bearing friction. These equations explicitly contain the empirical parameters of polytropic gas constant, discharge coefficients for both the main orifice and the snubber orifices, and the friction levels in the gear. These parameters are the only variables that appear in equations (01) and (02) that cannot be directly measured.

Equations (01) and (02) are highly nonlinear and are discontinuous due to the differing values of friction and discharge coefficient as a function of extension and compression. Future work will discuss more about the nature of these equations and present a method of solving these equations for gear displacements and velocities.

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