UPON EXPERIMENTAL DATA INTERPOLATION USING POWER FUNCTIONS. PART II: APPLICATION OF INTERPOLATION TO INDENTATION TESTS DATA

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Abstract: *The interpolation using power law functions for experimental data obtained from an indentation test, performed by a ball pressed on a metallic prismatic part is aimed both for loading and unloading curves. To pass up the errors induced by origin displacement, from the beginning, it was accepted the error and it will be found, together with all the other parameters, applying an optimization criterion. The accuracy of the method is proved by the value very close to 1.5 obtained for the exponent, related to the elastic unloading.*

Keywords: *experimental data, indentation test, interpolation*

1. Introduction

In the first part of the work, a number of features concerning theoretical considerations upon experimental data interpolation using power functions were presented. The main observation deduced from the first part of the paper is the strong influence that the identification of coordinate system’s origin where the experimental data are plotted upon the parameters of interpolation function. Next, several aspects concerning the interpolation of data from indentation tests are presented.

2. Theoretical consideration

One of the requirements of a quality mechanical design is the knowledge as accurate as possible, the physical and mechanical characteristics of the used materials. Among these properties, the elastic characteristics of the materials used in mechanical engineering play a major part. A first option is studying the technical literature. This method presents the advantage of low cost and promptness. The drawbacks of the method are, first, the fact that the exact composition of the material used is not known precisely and secondly, the fact that in catalogues, the value of mechanical characteristics lie in a domain, between inferior and superior limit. It is obvious that few designers would risk and when dimensioning a part, choose from tables, the upper admissible stress. Generally, dimensioning of parts is made using the lower admissible stress, also a safety coefficient and the results is supra-dimensioned part. To avoid this inconvenient, the domain within the aimed mechanical characteristic is positioned, should be narrowed. The ideal situation is to be able to establish experimentally the characteristics for the used material. One of the tests offering numerous information concerning mechanical characteristics of a material is the hardness test. As principle, the test consists in the analysis of the behaviour of a material when it is pressed using a punch or die. The simplest hardness test is the static experiment when the load carries out an increasing-decreasing cycle at very small velocities, and the information obtained are only the shape and dimensions of the plastic imprint. More data can be acquired when information concerning the force and deformation evolutions during the test is accessible. The force-deformation curve gives records regarding the two limit behaviours, perfectly elastic or perfectly plastic. To perform this characterization, under the assumption of a power law dependency between force and deformation, the power exponent is required. For the perfectly elastic behaviour, according to the Hertzian contact theory, [1], for the contact between two spheres, the exponent should be 3/2.

$$F = Cx^{\frac{3}{2}} \tag{1}$$

In the case that the materials does not present perfectly elastic behaviour, the value of the exponent decreases up to the value $\alpha = 1$, [2], that corresponds to the perfectly plastic comportment. Sneddon, [3, gave a reference paper where he found the relations between force and the deformation of an elastic half-space, pressed on the boundary surface by a rigid axially symmetric punch.

$$F = Ch^{\alpha} \tag{2}$$

where the power exponent takes the value $\alpha = 1$ for a flat face cylindrical punch, $\alpha = 3/2$ for the parabolic punch (limiting case for spherical punch) and $\alpha = 2$ for a conical punch.

Oliver and Pharr [4], established a technique for finding the Young modulus, methodology that assumes finding the slope at the beginning of unloading phase, supposed perfectly elastic, in the case of contact between an elastic body and a conical punch. The improvement of the method proposed by Oliver and Pharr resides in the small loading $(10^{-3} N)$ and deformation $(10^{-7} m)$ values, namely attaining nanoindentation range, and practically a non-destructive method.

Finding the correct exponent for force-deformation dependency is especially important constructing the dynamical models that describes the behaviour of systems with percussion. Lankarani proposes two models for the centric impact of two spheres, considering, first, that the entire kinetical energy variation is recovered as work of internal friction, [5], and secondly, as work of plastic deformation. For both models, the loading and unloading phases are described by power law functions.

3. Proposed method for data attainment from an experimental indentation test

In Fig. 1 is presented the loading-unloading curve for a contact between a bearing ball, *19mm* diameter, and the flat frontal face of a mild steel cylinder. As Prchlik emphasizes, [6], the most difficult task is to identify the point where actually the loading starts, as it can be observed from Fig. 2. The equipment used for the test, GADALBINI 600 machine form Materials Testing Laboratory, allows also recording the load and deformation variations with time.

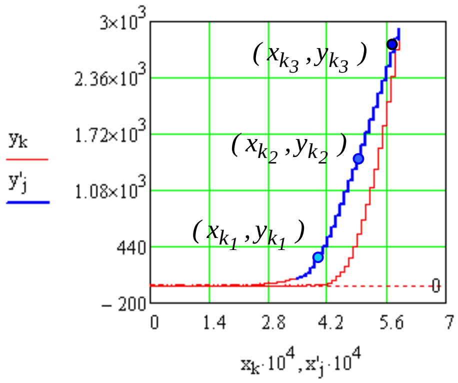

Figure 1 *Three characteristic points chosen from loading curve*

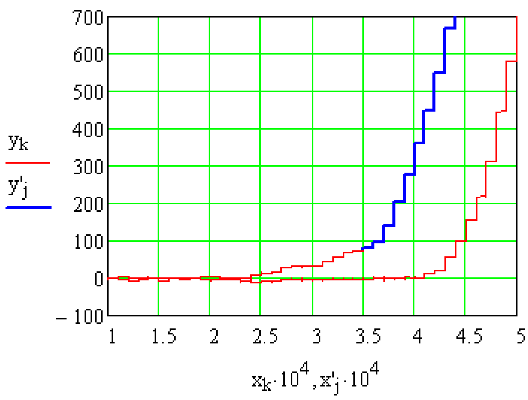

Figure 2 *Detail from a force-deformation curve for indentation test*

The complexity of point and moment of test's actual commencement is highlighted in Fig. 3 and Fig. 4 where there is presented the force-time dependency.

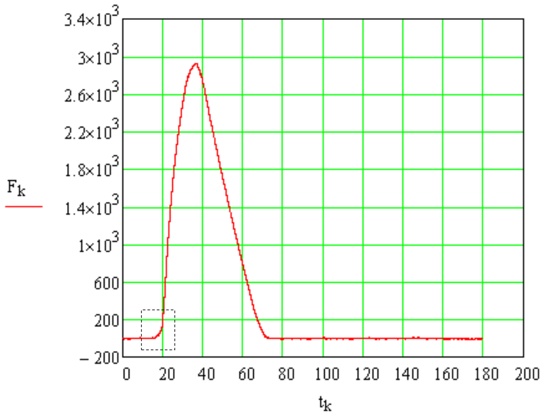

Figure 3 *Loading force versus time*

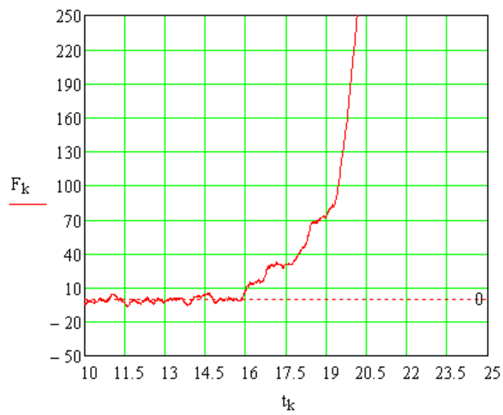

Figure 4 *Detail of loading force variation with time at the initiation of test*

One of the main causes influencing the difficulty of contact initiation identification is the discrepancy between the theoretical surfaces assumed smooth and the actual surfaces participating at contact. To support this affirmation, in Fig. 5 it is presented the indentation from an aluminium part after the impact with a steel ball. The plot attests that in initial phase, the contact between the micro roughness from the two surfaces is made firmly and after flattening these asperities, the certain contact between the two bodies takes place.

To surpass the difficulty concerning the contact initiation identification, one considers the observation that during the loading zone, Fig. 1, there are easily distinguished two regions: one, at start, when the force gradient is lower and the second, where the curve is steeper.

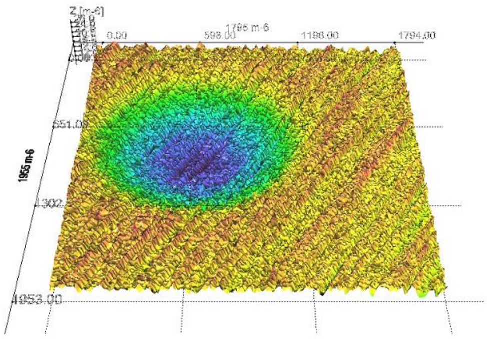

Figure 5 *Plastic imprint after an impact test*

The complexity consists in establishing precisely the point that separates the two zones. From the loading region, it is separated the zone where it is considered that the firm contact is made (blue coloured plot in Fig. 1 and Fig. 2). In this zone, is proposed a dependency having the form:

$$y = y_0 + C(x - x_0)^{\beta} \qquad (3)$$

where the constants C, , β x_0 and y_0 have to be found from the condition of minimum imposed to the function:

$$F(C,\beta,x_0,y_0) = \sum_{k_{in}}^{k_{fin}} [C(x_k - x_0)^{\beta} + y_0 - y_k]^2 \qquad (4)$$

The sum is extended for all the points with order numbers between an initial value k_{in} and k_{fin}, final value. Finding the actual values of the constants C, x_0 and y_0 assumes solving the system with four equations obtained by deriving the function from equation (4) with respect to each of the four parameters. The system obtained is a transcendental one and a numerical method is required for solving it.

Regardless of the methodology, the numerical procedure initiation supposes précising a guess value. A Newton - Raphson algorithm was applied for solving the system,

[7], but the method was inadequate because no mater the initial values considered, at some point, the functional determinant from the structure of the algorithm becomes zero and the procedure finishes.

Considering the facts presented in the first part of the paper, form Fig. 1, the relative error for finding the force for which the contact is accomplished is much smaller than the error for identification of deformation at firm contact installing. Specifically:

$$\frac{y_0}{|F_{max}-F_{min}|} << \frac{x_0}{|x_{max}-x_{min}|} \qquad (5)$$

One can consider $y_0 = 0$ and therefore the interpolation function becomes:

$$y = C(x-\xi_0)^{\beta} \qquad (6)$$

The values of the parameters from the interpolation function result from the minimum condition imposed to the function:

$$F(C,\beta,\xi_0) = \sum_{k_{in}}^{k_{fin}} [C(x_k-\xi_0)^{\beta} - y_k]^2 \qquad (7)$$

The actual form of the system:

$$\begin{cases} F_C = \dfrac{\partial F(C,\beta,\xi_0)}{\partial C} = 0 \\ F_\beta = \dfrac{\partial F(C,\beta,\xi_0)}{\partial C} = 0 \\ F_{\xi_0} = \dfrac{\partial F(C,\beta,\xi_0)}{\partial C} = 0 \end{cases} \qquad (8)$$

being, explicitly:

$$\begin{cases} \sum\limits_{k_{in}}^{k_{fin}} [C(x_k-\xi_0)^{\beta} - y_k](x_k-\xi_0)^{\beta} = 0 \\ \sum\limits_{k_{in}}^{k_{fin}} [C(x_k-\xi_0)^{\beta} - y_k](x_k-\xi_0)^{\beta} \ln(x_k-\xi_0) = 0 \\ \sum\limits_{k_{in}}^{k_{fin}} [C(x_k-\xi_0)^{\beta} - y_k](x_k-\xi_0)^{\beta-1} = 0 \end{cases} \qquad (9)$$

Again, a guess value solution is necessary to solve the system (9). To find a start solution as close as possible to the exact solution, three points from the considered zone are chosen, $(x_{k_1}, y_{k_1}), (x_{k_2}, y_{k_2}), (x_{k_3}, y_{k_3})$, two positioned in the vicinity of the extremities and one, in the centre of the zone, and the curve (6) is obliged to pass through the three points. Thus, it results the following system:

$$\begin{cases} y_{k_1} = C'(x_{k_1}-\xi'_0)^{\beta'} \\ y_{k_2} = C'(x_{k_2}-\xi'_0)^{\beta'} \\ y_{k_2} = C'(x_{k_2}-\xi'_0)^{\beta'} \end{cases} \qquad (10)$$

The solution of the above system, (C', β', ξ'_0) will be applied as guess value for solving the system (8) by Newton – Raphson algorithm. The advantage of employing the system (10) consists in the fact that the equations of the system can be solved with respect to C' and β'; finally, a transcendental equation with respect to ξ'_0 is obtained:

$$\frac{\ln\dfrac{y_{k_1}}{y_{k_3}}}{\ln\dfrac{y_{k_2}}{y_{k_3}}} = \frac{\ln\dfrac{x_{k1}-\xi'_0}{x_{k3}-\xi'_0}}{\ln\dfrac{x_{k2}-\xi'_0}{x_{k3}-\xi'_0}} \qquad (11)$$

and solving it presents no difficulty. It was observed that the solutions of system (8) are strongly dependant on the dimensions and position of the restriction from Fig. 1.

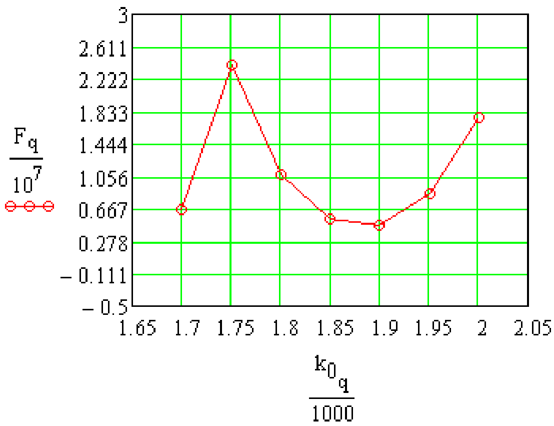

Figure 6 *The values of objective function for different values of initial deformation*

In Fig. 6 it is presented the variation of the function $F(C,\beta,\xi_0)$ having the parameters found by the methodology described above, considering that the region of appliance are the points starting with k_q and ending to the point where the force is maximum, corresponding to n.

For this value, In Fig. 7, are presented the experimental data, the interpolation curve and the curve (6) traced with the values obtained from system (10), specifically, for the guess values :

$$C' = 2.815 \cdot 10^8$$
$$\beta = 1.377$$
$$\xi_0 = 3.54 \cdot 10^{-4}$$

and the resulting exact solutions:

$$C = 2.744 \cdot 10^8$$
$$\beta = 1.376$$
$$\xi_0 = 3.488 \cdot 10^{-4}$$

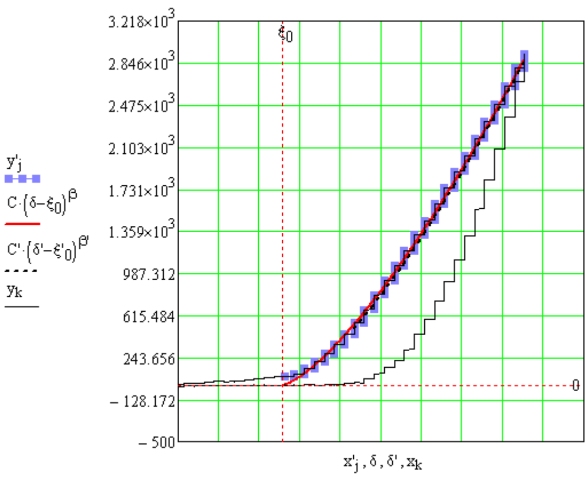

Figure 7 *Experimental data, interpolation curve for loading phase*

It can be observed that the exact solution is very close to the guess value.

Fig. 8 presents the results of the method applied for the unloading phase of the contact.

For the guess values :

$$C' = 7.617 \cdot 10^8$$
$$\beta' = 1.405$$
$$\xi'_0 = 4.513 \cdot 10^{-4}$$

the resulting exact solutions are:

$$C = 1.423 \cdot 10^9$$
$$\beta = 1.48$$
$$\xi_0 = 4.486 \cdot 10^{-4}$$

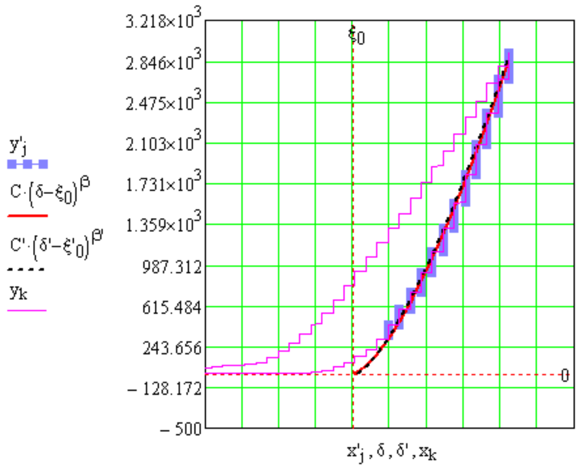

Figure 8 *Experimental data and interpolation curve for unloading phase*

The rightness of the method is validated by the value of the exponent found for the unloading phase, during which the material's behaviour is perfectly elastic, with $\beta = 1.5$, and the value obtained by experimental way is $\beta = 1.48$. An immediate application of the method is accurate finding of final plastic deformation.

$$x_{rem} = \xi_0^{unload} - \xi_0^{load} = 4.486 \cdot 10^{-4} - 3.488 \cdot 10^{-4} = $$
$$= 0.998 \cdot 10^{-4}\, m$$

Establishing the depth of the plastic imprint resulting after the contact is troubled by the pile-up effect that makes difficult the identification oz zero line of the profile.

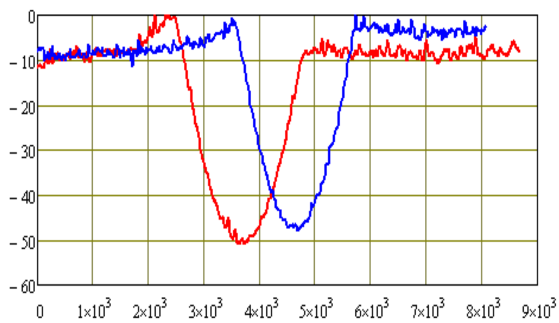

Figure 9 *Pile-up effect example*

For example, the radial and tangential profiles of an imprint obtained with a rotating disk collided by a bearing ball in free fall, are presented in Fig. 9, [8]

3. Conclusions

In the first part of the paper, via an example, it was proved the effect of origin displacement upon accuracy of finding the parameters of interpolation power function. The present part of the work, considers the experimental data obtained from an indentation test, performed by a ball pressed on a metallic prismatic part. It is aimed the interpolation using power law functions both for loading and unloading curves. To avoid the errors induced by origin displacement, the interpolation functions accepts from the beginning that this error exists and it will be found, together with all the other parameters, applying an optimization criterion. The correctness of the methodology is validated by the value very close to 1.5 obtained for the exponent, value corresponding to an elastic unloading, as engineering literature reveals.

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