THE SURFACE CONTACT OF VISCOELASTIC MATERIALS. PART II – RESULTS AND DISCUSSIONS

Sergiu Spinu[1,2], Delia Cerlinca[1,2]

[1] *Department of Mechanics and Technologies, Stefan cel Mare University of Suceava, 13th University Street, 720229, Romania, e-mail: sergiu.spinu@fim.usv.ro, delia@fim.usv.ro*
[2] *Integrated Center for Research, Development and Innovation in Advanced Materials, Nanotechnologies, and Distributed Systems for Fabrication and Control (MANSiD), Stefan cel Mare University, Suceava, Romania.*

Abstract: *The contact problem of a rigid flat–ended indenter with a curvature at the edge, pressed against a linear viscoelastic half-space, is simulated numerically via an algorithm proposed in a companion paper. The imposed viscoelastic behavior is based on experimental measurements reported in the literature, fitted to a two-term Prony series, thus having two relaxation times. The creep compliance data is derived from the relaxation modulus based on the reciprocal relation between the two functions in the Laplace transform domain. Three representative contact geometries are considered, and load eccentricity is also allowed. The tilting angle and the intensity of the maximum pressure during the loading history are assessed. The performed simulations suggest the ability of the numerical program to predict the behavior of viscoelastic contact processes involving complex rheological models and diverse contact geometry.*

Keywords: *numerical simulation, viscoelastic surface contact, indentation, eccentric loading*

1. Introduction

Contacts involving nominally flat surfaces appear often in engineering applications such as supporting feet/pads or electrical brushes. The solution usually adopted for the flat axisymmetrical indentation is that of Boussinesq, i.e. of a rigid indenter pressed into a compliant elastic half-space [1,2], in which the pressure distribution is singular at the edge. To relieve the singularity, which can be considered unrealistic in practice, Ciavarella [3] studied the effect of adding a finite curvature at the indenter edge. These authors [4] also studied numerically the effect of profiling on the pressure distribution achieved in a surface axisymmetrical contact.

Due to their structural complexity, viscoelastic materials are now widely used in many engineering fields. The analytical description of a surface contact involving viscoelastic materials may be carried out provided the contact problem is axisymmetric [5-7], although mathematical complexity may impede the wide range application of the aforementioned analytical solutions.

The numerical approach has proven itself as a versatile tool for contact analysis, capable of extending the limited analytical results, and thus advancing the prediction capability and the understanding of the contact process. An algorithm proposed in a companion paper [8] is employed herein to study by means of numerical simulation the flat-ended contact of linear viscoelastic materials.

2. Viscoelastic behavior

The textile, petroleum, automobile, and pharmaceutical industries make extensive use of polymeric materials, whose viscoelastic properties define their mechanical performance, as well as the achievement of successful processing methods at intermediate stages of production. Due to their simplicity and generality, classic rheological models (e.g. the Maxwell, or the Kelvin units) are often used for the characterization of the viscoelastic behavior. However, these basic models, having only one relaxation time, are susceptible to provide a qualitative description of the viscoelastic behavior, whereas quantitative assessments requires more parameters, related to the naturally occurring spectrum of relaxation times of the investigated viscoelastic material. Such a goal can be accomplished by using a complex model such as the Wiechert model, which consists in several Maxwell units and a free spring, connected in parallel. The shear relaxation modulus function of the Wiechert model can be expressed as [9]:

$$\Psi(t) = g_\infty + \sum_{i=1}^{n} g_i \exp\left(\frac{-t}{\tau_i}\right), \qquad (1)$$

where g_∞ is the long term modulus once the material is totally relaxed, and τ_i and g_i, with $\tau_i = \eta_i / g_i$, the relaxation time and the spring stiffness of each Maxwell unit, as shown in Fig. 1. The naturally occurring spectrum of relaxation times of a viscoelastic material can be described by including as many exponential terms as needed. Relation (1) is also referred to as a Prony series. The Prony series of a viscoelastic material is usually obtained by a one-dimensional relaxation test, in which the viscoelastic material is subjected to a sudden strain which is kept constant during the test, while measuring the stress response over time. The initial stress is related to the purely elastic response of the material. Later on, the stress relaxes due to the viscous effects in the viscoelastic material. Mathematical description by the Prony series can be achieved by fitting the experimental data to Eq. (1), by adjusting the model parameters g_∞, g_i and τ_i.

The constitutive law of the viscoelastic material employed in this paper is that of the polymethyl methacrylate (PMMA), a thermoplastic polymer whose mechanical properties were studied extensively by Kumar and Narasimhan [10]. The latter authors obtained experimentally the PMMA relaxation modulus data under uniaxial compression in a window of observation of 1000 s. Based on their results, the two-term Prony series of PMMA results as:

$$\Psi(t) = \begin{bmatrix} 1973 + 254.776 \cdot \exp\left(\frac{-t}{8.93}\right) + \\ 263.6628 \cdot \left(\frac{-t}{117.96}\right) \end{bmatrix}, \text{[MPa]}, \qquad (2)$$

which is the modulus relaxation function of the material, depicted in Fig. 2.

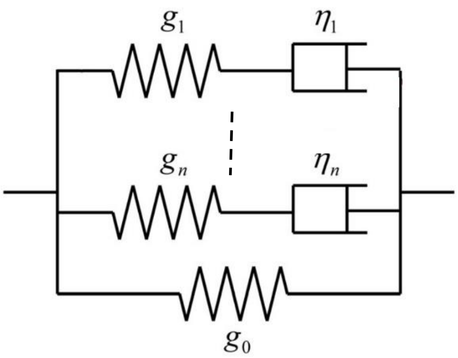

Figure 1: The Weichert model

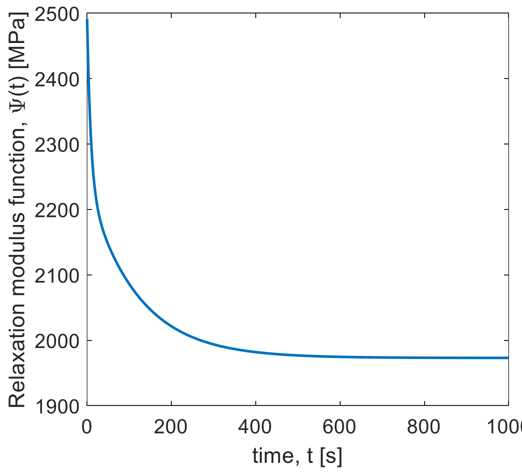

Figure 2: Relaxation modulus function of PMMA

In the time domain, the creep compliance and relaxation modulus functions are not reciprocal, i.e. $\Psi(t)\Phi(t) \neq 1$, but in the Laplace transform domain, the following relation applies [11] to their transforms:

$$\bar{\Psi}(s)\bar{\Phi}(s) = 1/s^2, \qquad (3)$$

where s is the variable in the Laplace transform domain. The latter equation can be used to derive the creep compliance of PMMA, by computing its Laplace transform first from Eq. (3), and by subsequently applying inverse Laplace transform to obtain $\Phi(t)$ in the time domain:

$$\Phi(t) = 7\cdot 10^{-4} - 6.17\cdot 10^{-5}\exp(-0.1t) - 8.38\cdot 10^{-5}\exp(-7.47\cdot 10^{-3}t),\ [1/\mathrm{MPa}]. \tag{4}$$

3. Reference simulation

The reference simulation consists in a flat axisymmetrical (i.e. circular) indenter having a finite curvature at the boundary. The assumption of an abrupt edge leads [2] to a pressure distribution having a singularity at the edges, which can be unrealistic. In practice, the rounding of indenter edge can result from: (1) limitations of precision in the manufacturing processes, or (2) localized plasticity during the first loadings. The results is, in both cases, the relieve of the pressure singularity.

The effects of the rounding radius in the elastic axisymmetric indentation of a flat surface was studied numerically by these authors [4] using a FFT-based numerical program, whereas these authors [12] advanced the generalization of the contact code to allow for eccentric loading.

This paper extends the results by computing the viscoelastic response of a PMMA half-space loaded by a rigid flat-ended circular indenter having a rounding radius at the edge. The axisymmetrical elastic solution for this contact scenario was derived by Ciavarella [3], who obtained the relation between the contact radius and the normal load transmitted through the contact.

The contact geometry, depicted in Fig. 3, is described in radial coordinates by the following equation:

$$hi(r) = \begin{cases} 0, & 0 \le r \le R_a, \\ \dfrac{1}{2R_c}(r - R_a)^2, & R_a \le r. \end{cases} \tag{5}$$

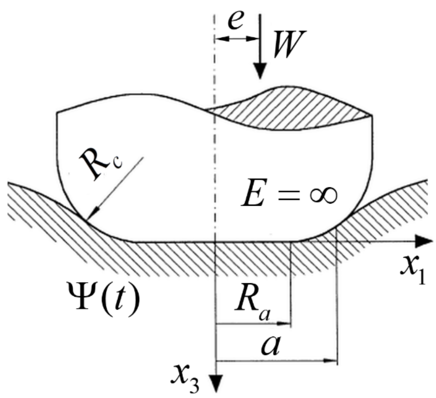

Figure 3: The contact geometry

In the elastic solution [3], the relation between the applied load W and the contact radius a can be expresses as:

$$W = \frac{R_a^3}{3R_c\eta}\frac{3\sin\phi + \sin^3\phi - 3\phi\cos\phi}{\cos^3\phi},\ \cos\phi = \frac{R_a}{a}, \tag{6}$$

where η denotes the contact compliance. This relation holds as long as the contact disk does not enter far into the rounded off region, i.e. a does not exceed $R_a + R_c$. The latter assumption was also kept in the viscoelastic contact simulations performed in the following section. The contact radius a at time $t = 0$ and the average pressure $p_m = W/(\pi a^2)$ were used as normalizers. The dimensionless load eccentricity was defined as $e = x_1'/R_a$.

4. Viscoelastic indentation

In a first set of simulations, the load was applied centrically and the geometry of the indenter was varied under the constraint that $R_a + R_c = a$. Three cases were considered: (1) $R_a = 0.2a$, (2) $R_a = 0.2a$, and (3) $R_a = 0.5a$. The load was set according to Eq. (6), assuring that the same contact radius is achieved in all three cases. The predicted pressure distributions for specific times in the loading history, normalized by the average pressure p_m, are depicted in Figs. 4-6. These distributions suggest that both central and maximum pressure decrease with time, whereas the opposite is true for the contact radius.

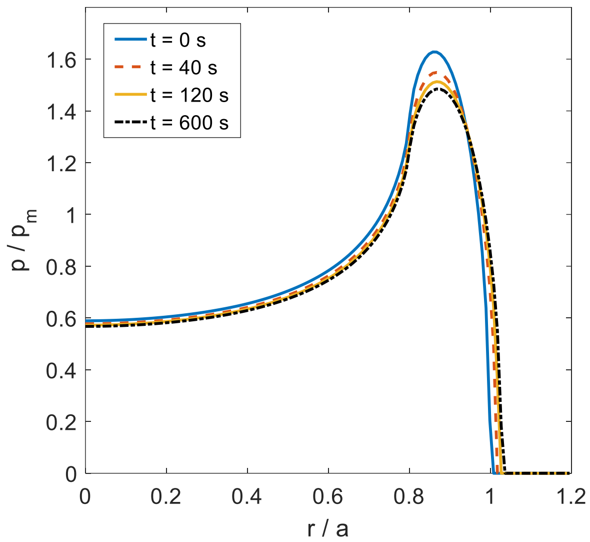

Figure 4: Pressure history, $R_a = 0.2a$

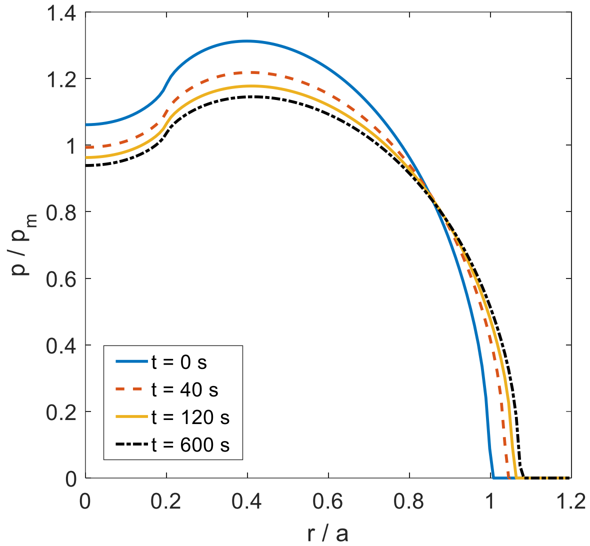

Figure 5: Pressure history, $R_a = 0.8a$

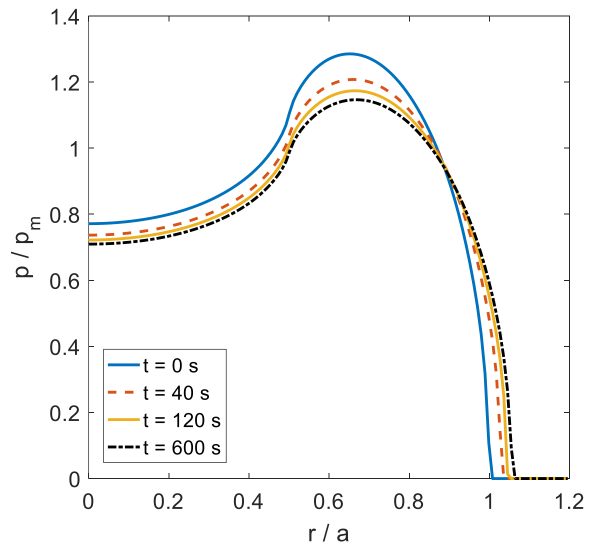

Figure 6: Pressure history, $R_a = 0.5a$

The variations of the contact area and of the maximum and central pressure during the observation window are displayed in Figs. 7 and 8, respectively. The values of the corresponding parameters in the beginning of the simulation window are used as normalizers. Numerical simulations suggest that, the smaller the radius of the flat zone of the indenter, the greater the increase in the contact area over time. The decrease in the central and maximum pressure follows the same trend.

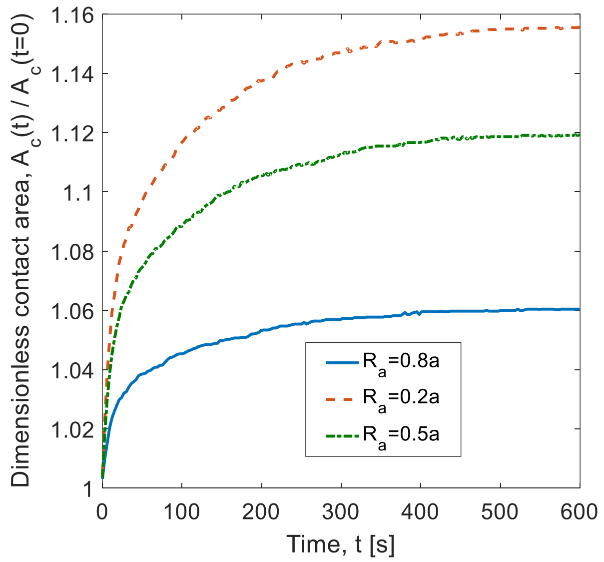

Figure 7: Contact area vs. time

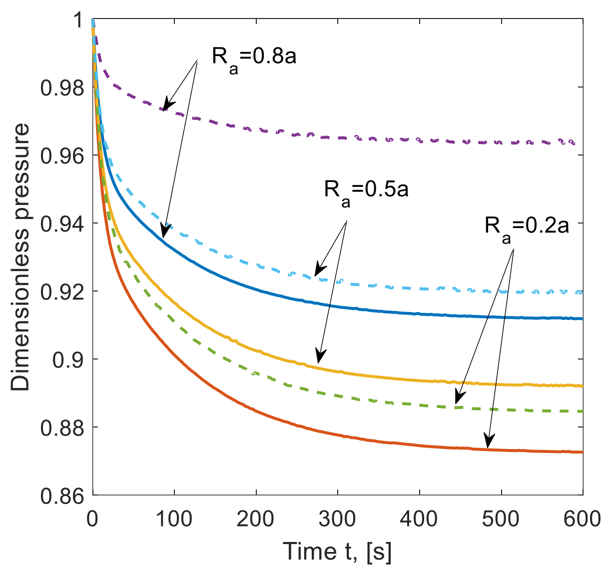

Figure 7: Dimensionless maximum (continuous lines) and central (dashed lines) pressure vs. time

5. Eccentric loading

The case (3) from the previous simulations was kept as reference and the load dimensionless eccentricity was varied between 0.1 and 1. The predicted pressure histories are depicted in figures 9-12. In all cases, the centric pressure distribution corresponding to $t = 0$ is also plotted for reference. The history of the dimensionless tilting angle is displayed in Fig. 13, whereas Fig. 14 shows the variation of the maximum pressure normalized by the medium pressure. In all cases, the maximum pressure decreases with time.

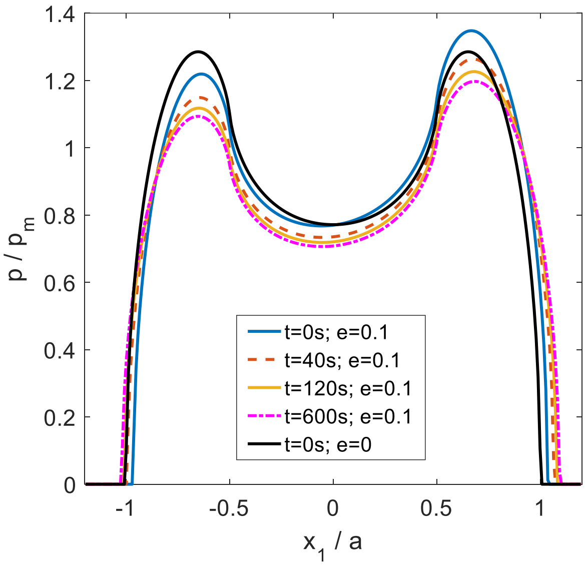

Figure 9: Pressure history, $e = 0.1$

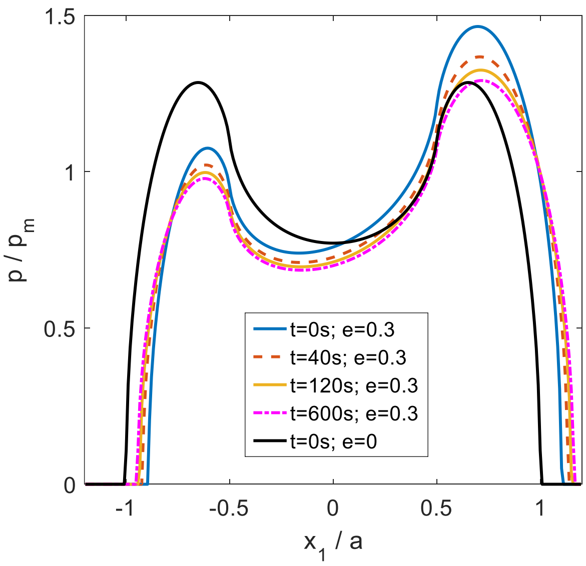

Figure 10: Pressure history, $e = 0.3$

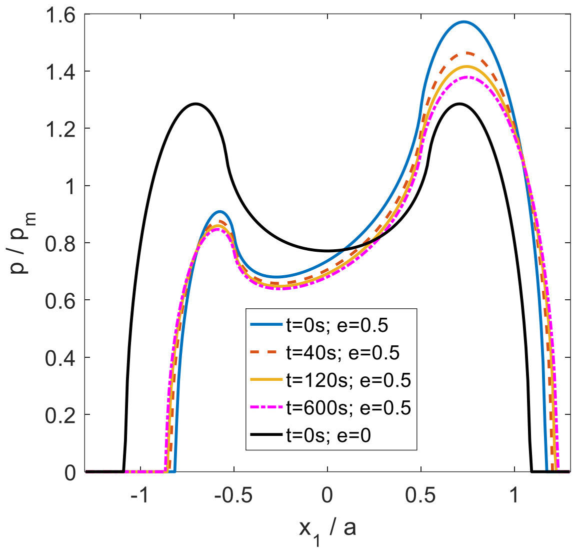

Figure 11: Pressure history, $e = 0.5$

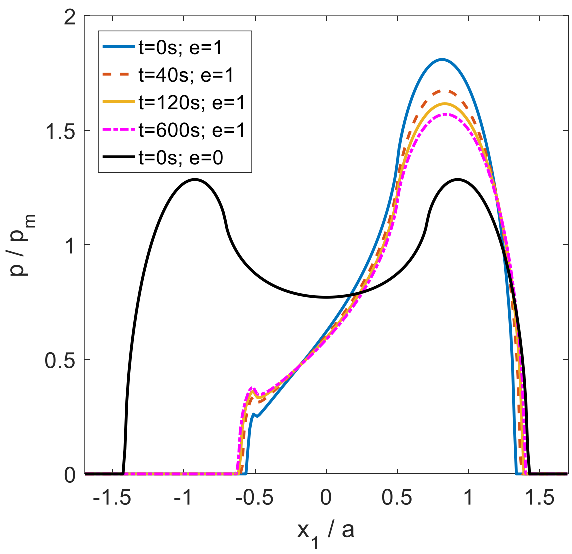

Figure 12: Pressure history, $e = 1$

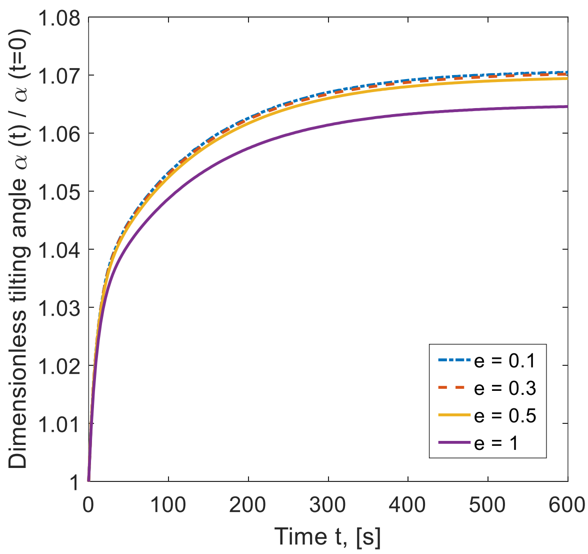

Figure 13: Dimensionless tilting angle vs. time

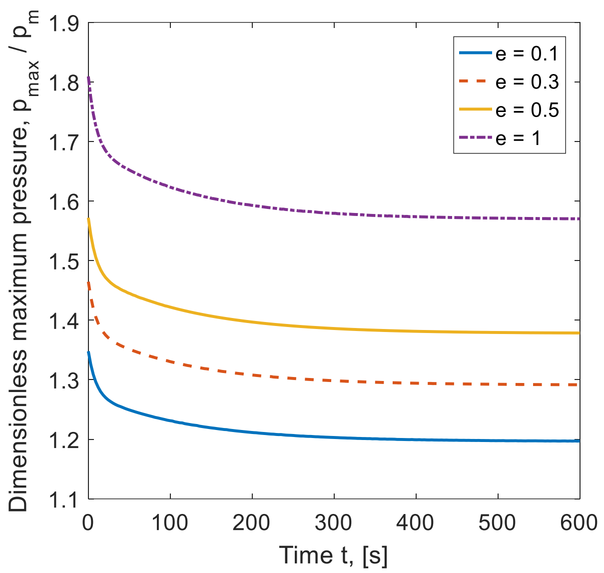

Figure 14: Dimensionless maximum pressure vs. time

6. Conclusions

A previously proposed computer program for the simulation of conforming viscoelastic contacts is used in this paper to study the indentation of a polymethyl methacrylate half-space by a rigid, flat-ended, axisymmetrical indenter with rounded corners. The linear viscoelastic rheological behavior of the material is described by a Wiechert model, expressed mathematically as a two-term Prony series, whose parameters were derived from experimental measurements presented in the literature for a PMMA specimen undergoing uniaxial compression.

The indenter geometry is modified to account for the influence of profiling on the contact parameters. It was found that, the smaller the radius of the indenter flat region, the more pronounced the increase in the contact area, as well as the decrease of both maximum and central pressure over time.

Compared to existing pseudo-analytical formulations, the newly advanced computer program can extend the results to contact problems that are not axisymmetrical, e.g. the conforming contact under eccentric loading. The influence of load eccentricity on the history of the tilting angle and of the maximum contact pressure is assessed. It is found that the tilting angle increases with time.

The conducted numerical simulations prove that the newly advanced computer program can simulate a large range of contact processes involving linear viscoelastic materials.

7. Acknowledgement

This work was partially supported from the project “Integrated Center for Research, Development and Innovation in Advanced Materials, Nanotechnologies, and Distributed Systems for Fabrication and Control”, Contract No. 671/09.04.2015, Sectoral Operational Program for Increase of the Economic Competitiveness co-funded from the European Regional Development Fund.

8. References

[1] Shtaerman, I. Ya., 1949, *Contact Problem of the Theory of Elasticity*, Moscow.
[2] Timoshenko, S. P., and Goodier, J. N., 1970, *Theory of Elasticity*, 3rd ed., McGraw-Hill, New York.
[3] Ciavarella, M., 1999, *Indentation by Nominally Flat or Conical Indenters with Rounded Corners*, Int. J. Solids.Struct., **36**, pp. 4149-4181.
[4] Spinu, S., Gradinaru, D., and Marchitan, M., 2006, *FFT Analysis of Elastic Non-Hertzian Contacts–Effect of Rounding Radius upon Pressure Distribution and Stress State*, Proc. of VAREHD13-13th Conference on EHD Lubrication and Traction, Suceava, Romania.
[5] Ting, T. C. T., 1966, *The Contact Stresses Between a Rigid Indenter and a Viscoelastic Half-Space*, ASME J. Appl. Mech., **33**(4), pp. 845-854.
[6] Ting, T. C. T., 1968, *Contact Problems in the Linear Theory of Viscoelasticity*, ASME J. Appl. Mech., **35**(2), pp. 248-254.
[7] Greenwood, J. A., 2010, *Contact between an Axisymmetric Indenter and a Viscoelastic Half-Space*, Int. J. Mech. Sci., **52** (6), p. 829.
[8] Spinu, S., and Cerlinca, D., 2017, *The Surface Contact of Viscoelastic Materials.*

Part I – Algorithm Overview, submitted to Tehnomus Journal.

[9] Brinson, H. F., and Brinson, L. C., 2008, *Polymer Engineering Science and Viscoelasticity: An Introduction*, Springer, New York.

[10] Chen, W. W., Wang, Q. J., Huan, Z., and Luo, X., 2008, *Semi-Analytical Viscoelastic Contact Modeling of Polymer-Based Materials*, ASME J. Tribol., **133**(4), 041404 (10 pages).

[11] Kumar, M. V. R., and Narasimhan, R., 2004, *Analysis of Spherical Indentation of Linear Viscoelastic Materials*, Curr. Sci., **87**, pp. 1088-1095.

[12] Spinu, S., and Diaconescu, E., 2008, *Numerical Simulation of Elastic Conforming Contacts under Eccentric Loading*, Proc. of STLE/ASME 2008 International Joint Tribology Conference, pp. 649-651.